Precision Measurement of the Mean Curvature

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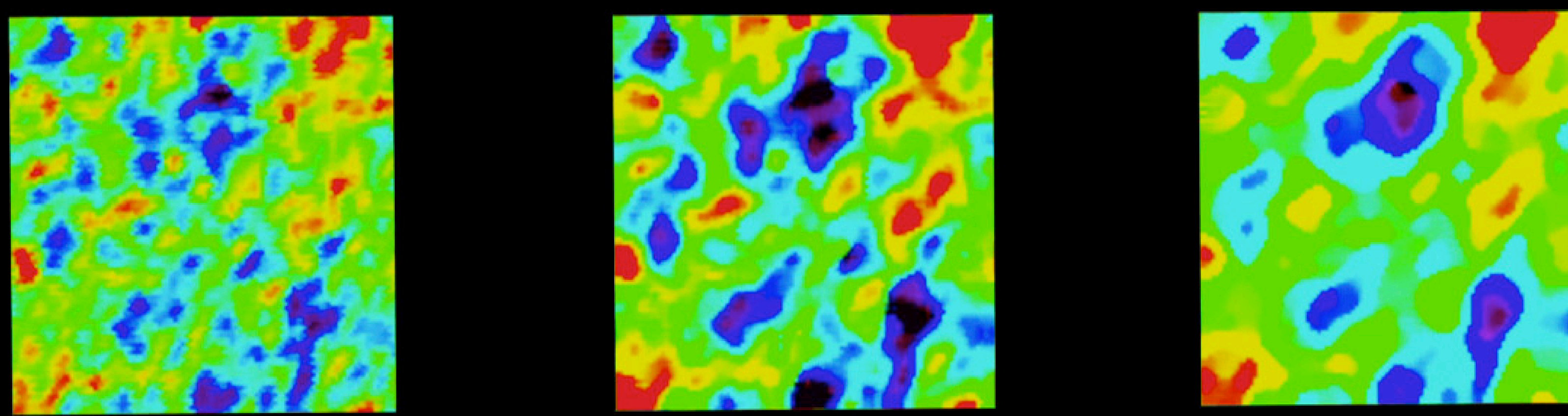
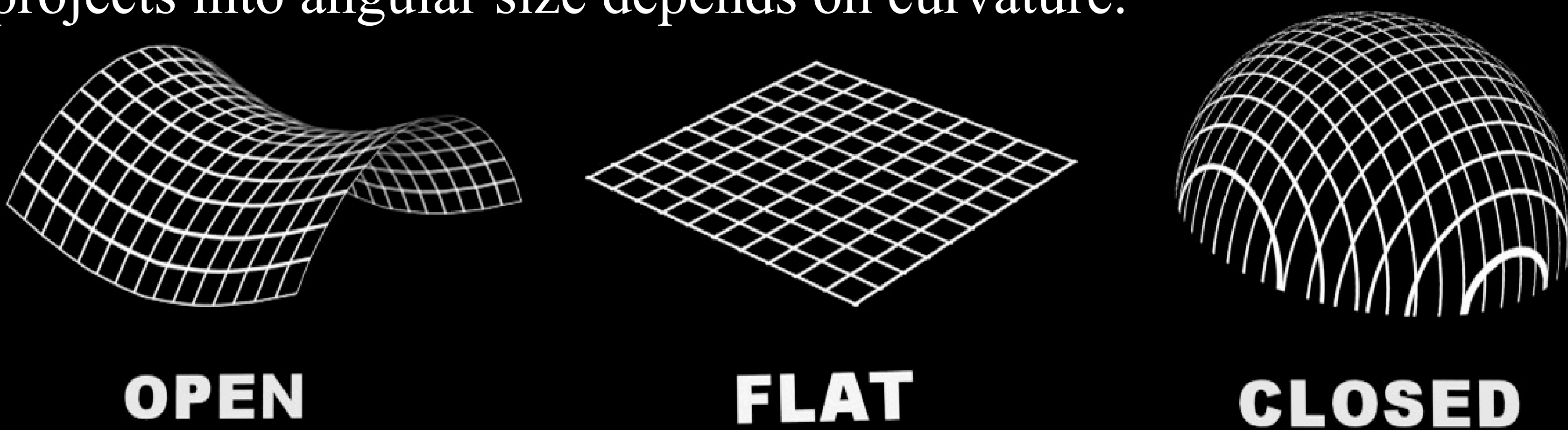
GEOMETRY OF THE UNIVERSE



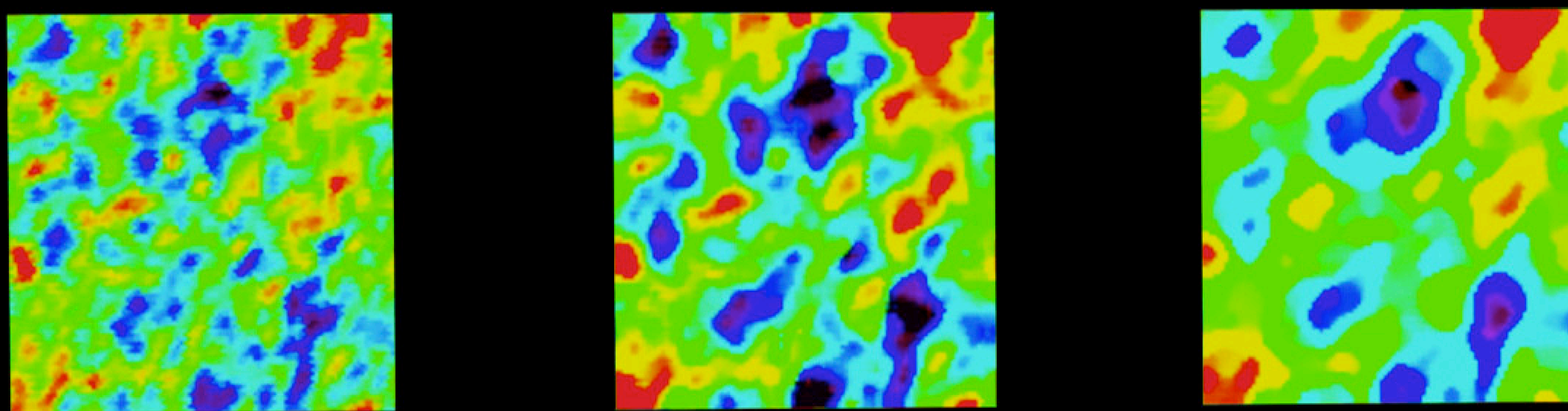
Physical size of typical hot/cold spot can be calculated. How this projects into angular size depends on curvature.



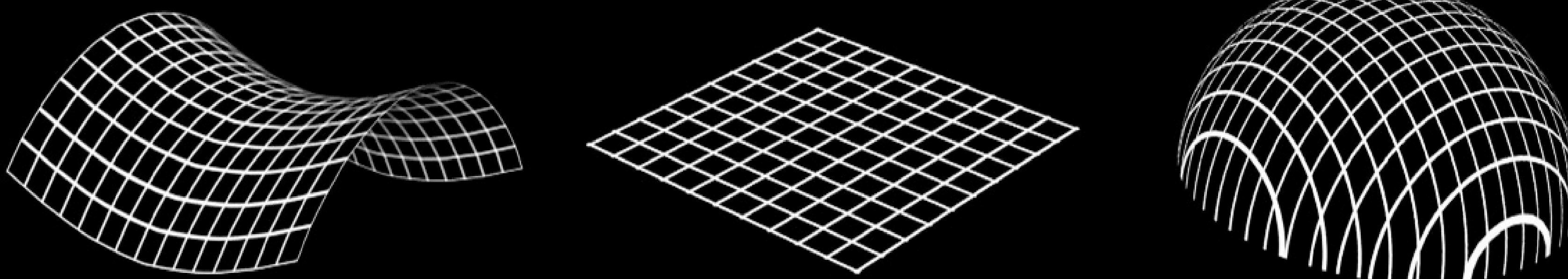
"Weighing the Universe with the CMB"

Jungman et al., Phys.Rev.Lett. **76**, 1007 (1996).

GEOMETRY OF THE UNIVERSE



Physical size of typical hot/cold spot can be calculated. How this projects into angular size depends on curvature and *comoving distance*.



So how has Ω_{tot} ' 1 been inferred from CMB data? Answer:

$\partial D_A/\partial\Omega_{tot}$ ' 5 $\partial D_A/\partial\Omega_\Lambda$ ➔ If $\sigma(\Omega_\Lambda) = 0.5$ then $\sigma(\Omega_{tot}) = 0.1$

Outline

- Importance of Mean Curvature Measurement
- Dark Energy / Curvature Degeneracy
- A Straightforward Solution
- Standard Candles and Standard Rulers
- Conclusion

Why Measure Mean Curvature?

- Robust Prediction of Inflation

$$<\rho>/\rho_c = 1 \text{ § } 10^{-60}$$

$<\rho>_H /\rho_c=1$ § 10^{-5} (averaged over Hubble patch)

- Probe of Fluctuations on Super-horizon Scales

How Well Is it Known Already?

- If we assume the dark energy is a cosmological constant, the SDSS baryon oscillation detection combined with CMB data gives a very impressive constraint of Ω_{tot} = 1.01§ 0.009

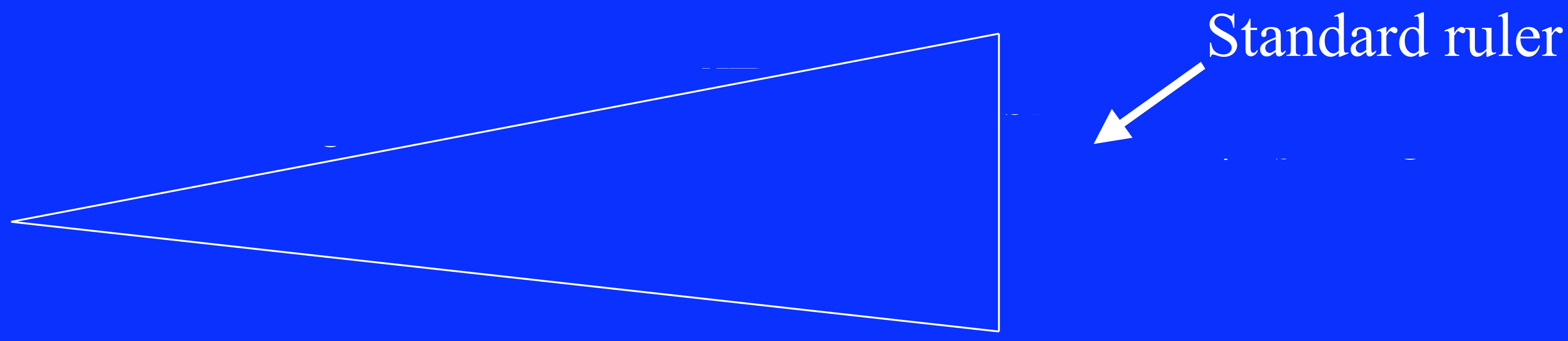
But we don’t know that the dark energy is a cosmological constant. You may have noticed there’s a minor effort underway to investigate the nature of the dark energy. If we allow w not equal to –1 then this constraint weakens considerably.

Dark Energy / Curvature Degeneracy

The comoving size of the sound horizon depends on matter density and baryon density, which can be inferred from CMB acoustic peak morphology, and thereby calibrated.

Measure θ and infer D_A

$\Delta D_A/D_A = \Delta r_s/r_s$ ' $-0.25\ \Delta\rho_m/\rho_m$



But D_A depends on both curvature and matter content ➔ degeneracy

Ω_Λ - Ω_k degeneracy: Eisenstein et al. (1998), Efstathiou and Bond (1999)

Dark Energy / Curvature Degeneracy

$$ds^2 = dt^2 - a^2(t)[dr^2/(1-kr^2) + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

From line element, $D_A = r$, and comoving distance from origin to r is

$$l = \int_0^r dr'/(1-kr'^2)^{1/2}$$

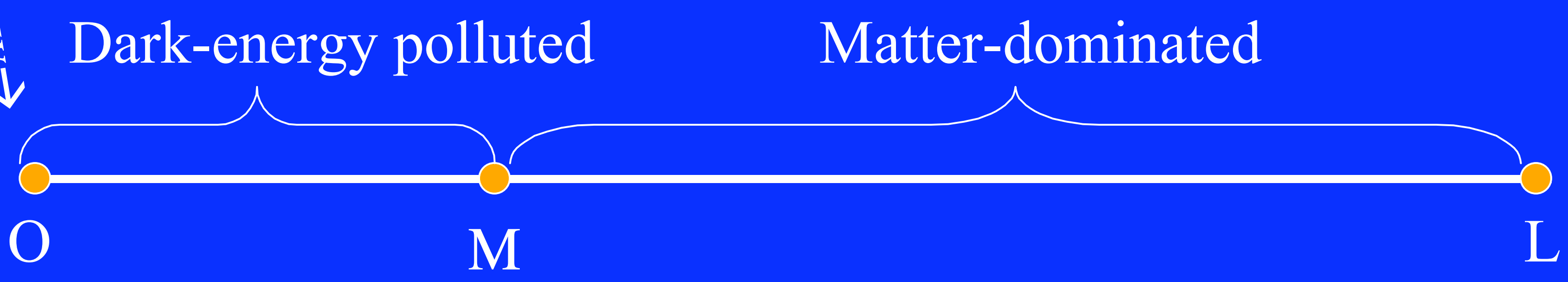
Solving for r to lowest order in k we have

$$D_A = r = l + kl^3/6$$

If we knew l and D_A we could solve for k. But we don't know l. Instead, we can calculate the comoving distance traveled by a photon that suffers a redshift, z:

$$l(z) = \int_0^z dz'/H(z') \quad \text{where } H^2(z) = 8\pi G\rho(z)/3 - k/a^2$$

Precision Determination of Mean Curvature



Measure D_{OL} (with CMB) and D_{OM} (e.g., baryon oscillations)

Calculate l_{ML} (given ρ_m from CMB)*

In absence of curvature, $D_{OL}-(D_{OM}+l_{ML}) = 0$

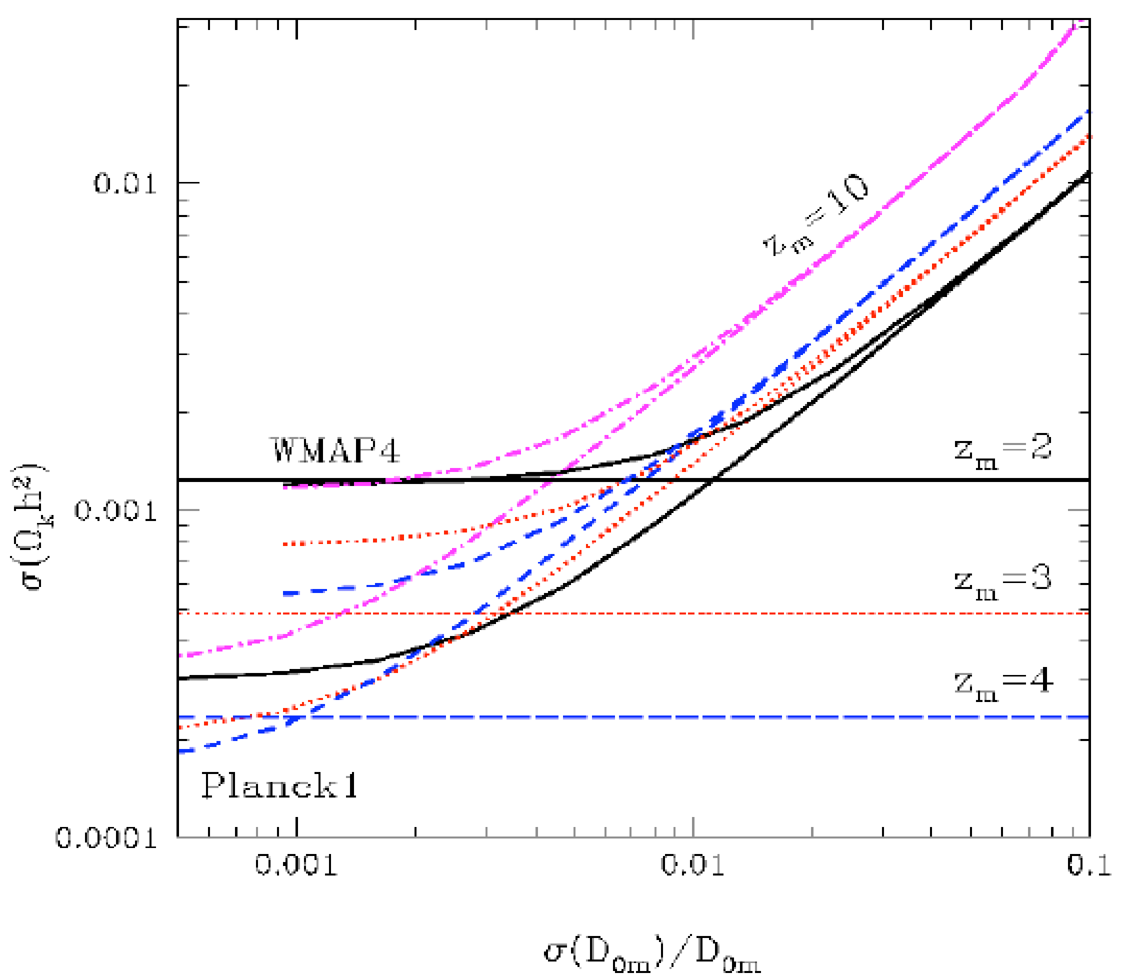
More generally (for $|\Omega_k| \ll 1$):

$$D_{OL}-(D_{OM}+l_{ML}) = \Omega_k H_0^2 (D_{OL}^3 - D_{OM}^3)/6$$

*Note: l_{ML} is the comoving distance, equal to angular diameter distance D_{ML} if $\Omega_k = 0$.

Error on Curvature Given Error on D_{OM}

$\Omega_k h^2 = (h/H_0)^2(D_{OL} - (D_{OM}+l_{ML})/(D_{OL}{}^3 - D_{OM}{}^3)$



Horizontal lines: bias in method due to dark energy at z > z_M

Other lines: error in Ω_k h^2 due to CMB errors on Ω_b h^2 and Ω_m h^2 as well as D_{OM} measurement error.

In limit of perfect D_{OM}, errors in D_{OL} and l_{ML} (due to error in ρ_m) partially cancel.

10^{-5} is difficult!

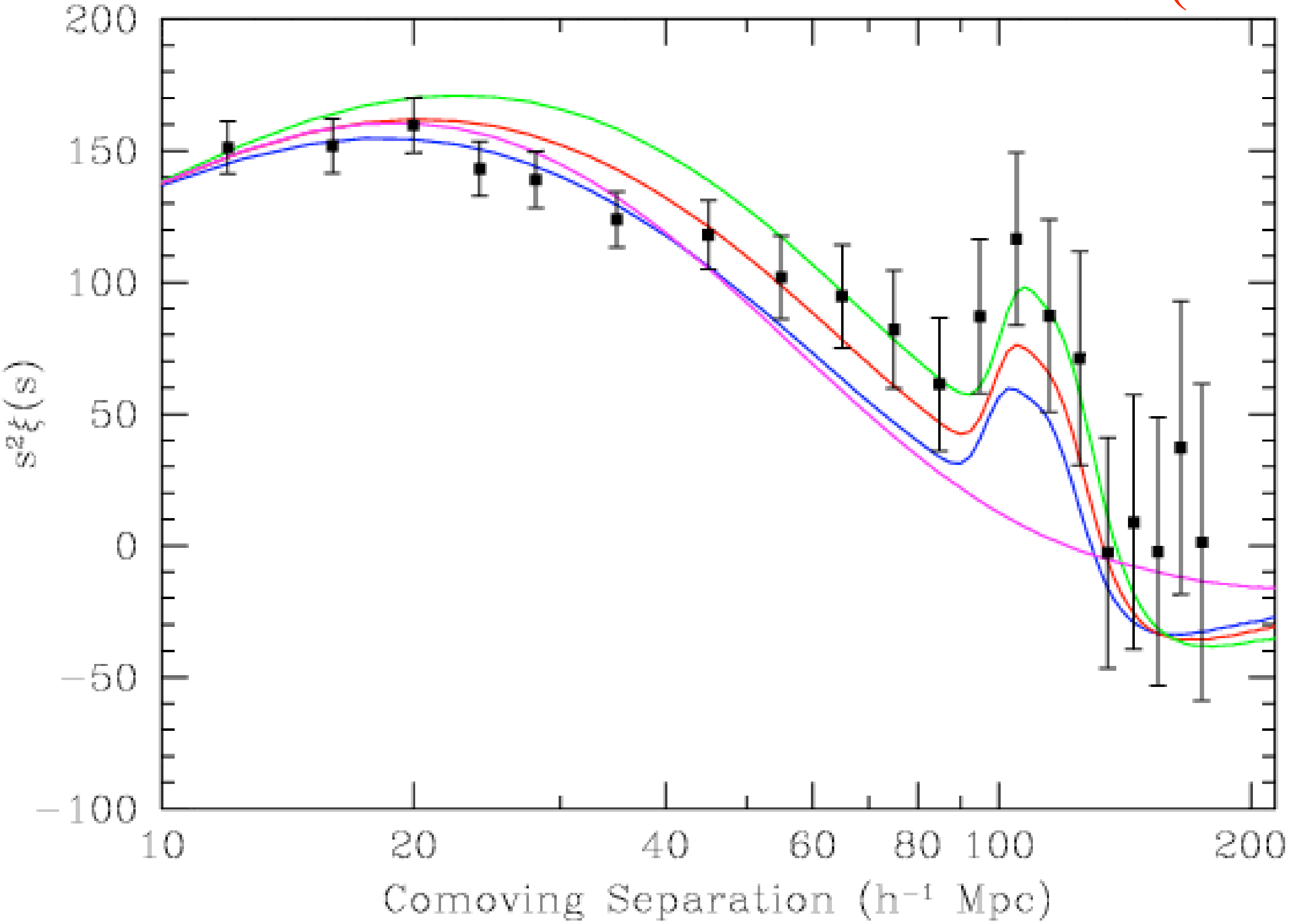
Measuring D_{OM}

- Standard Candles
  - SNeIa, GRB?, ??
- Standard Rulers in Matter Power Spectrum
  - sound horizon at last-scattering:
    r_s ➔ measure D_A/r_s
  - particle horizon at matter-radiation equality: /
    $1/\rho_{m,0}$ ➔ measure $D_A\ \rho_{m,o}$ (or $D_A\omega_m$).

$(\omega_m = \rho_{m,0}/\rho_c\ h^2)$

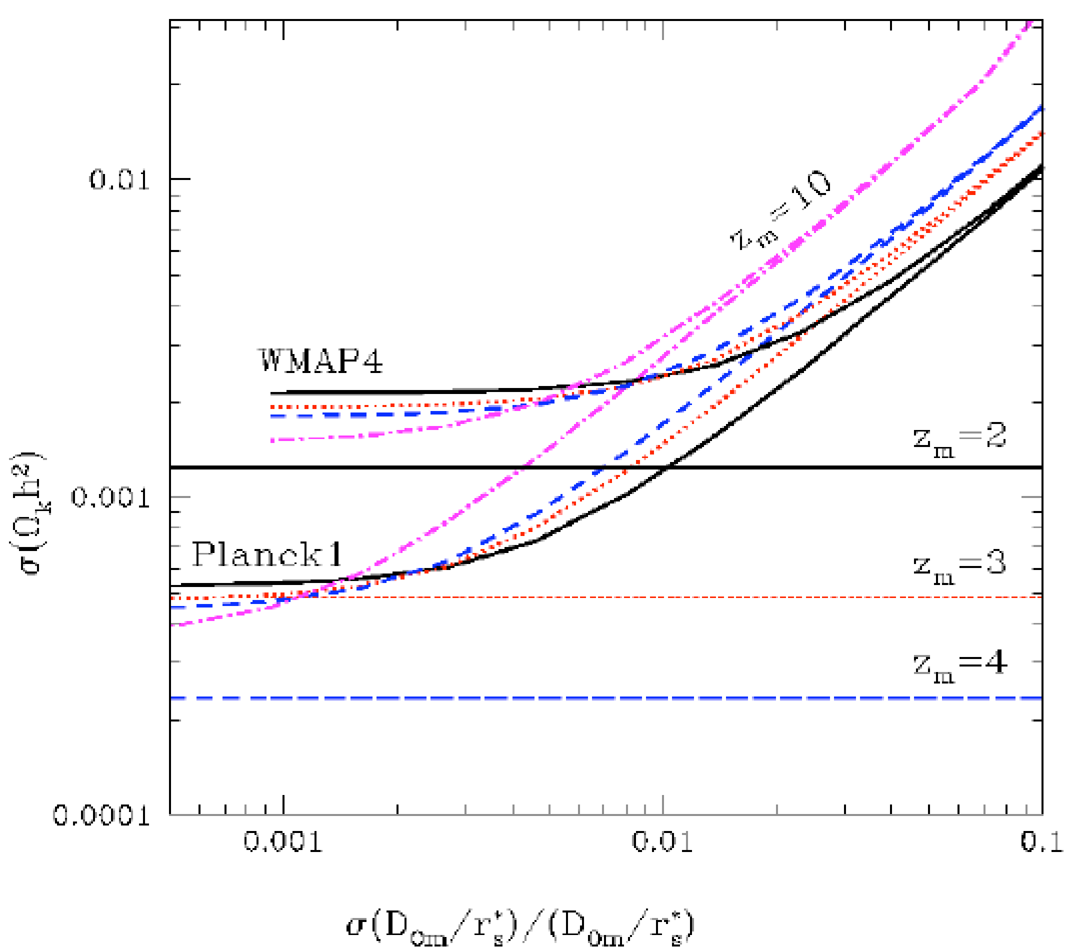
Correlation Function of SDSS Luminous Red Galaxies

Eisenstein et al. (2005)



Curvature Error Given Error on D_{OM}/r_s

$\Omega_k h^2 = 6(h/H_0)^2 r_s^{-2} (D_{OL}/r_s – (D_{OM}/r_s + l_{ML}/r_s))/((D_{OL}/r_s)^3 – (D_{OM}/r_s)^3)$

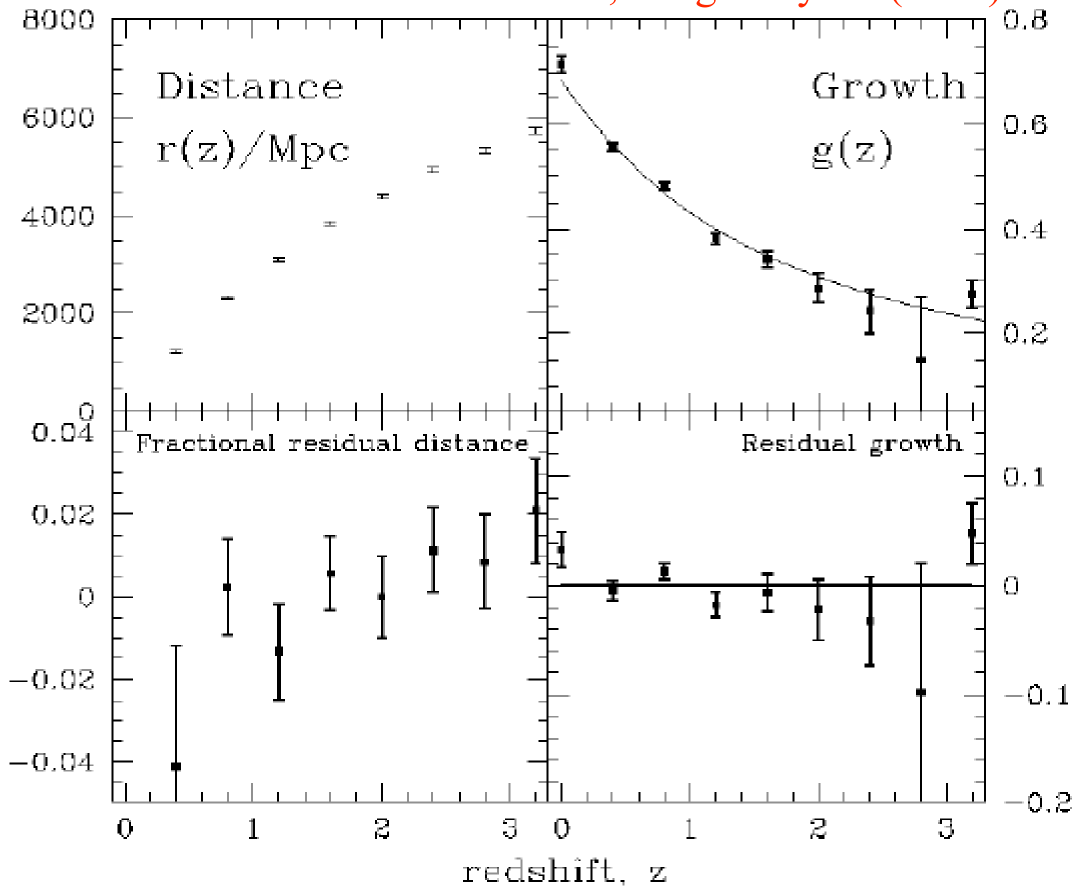


No significant error in D_{OL}/r_s (=$1/\theta_s$).

In limit of perfect D_{OM}, error is entirely from l_{ML} error.

Distance (and growth) reconstructed from LSST WL survey + Planck

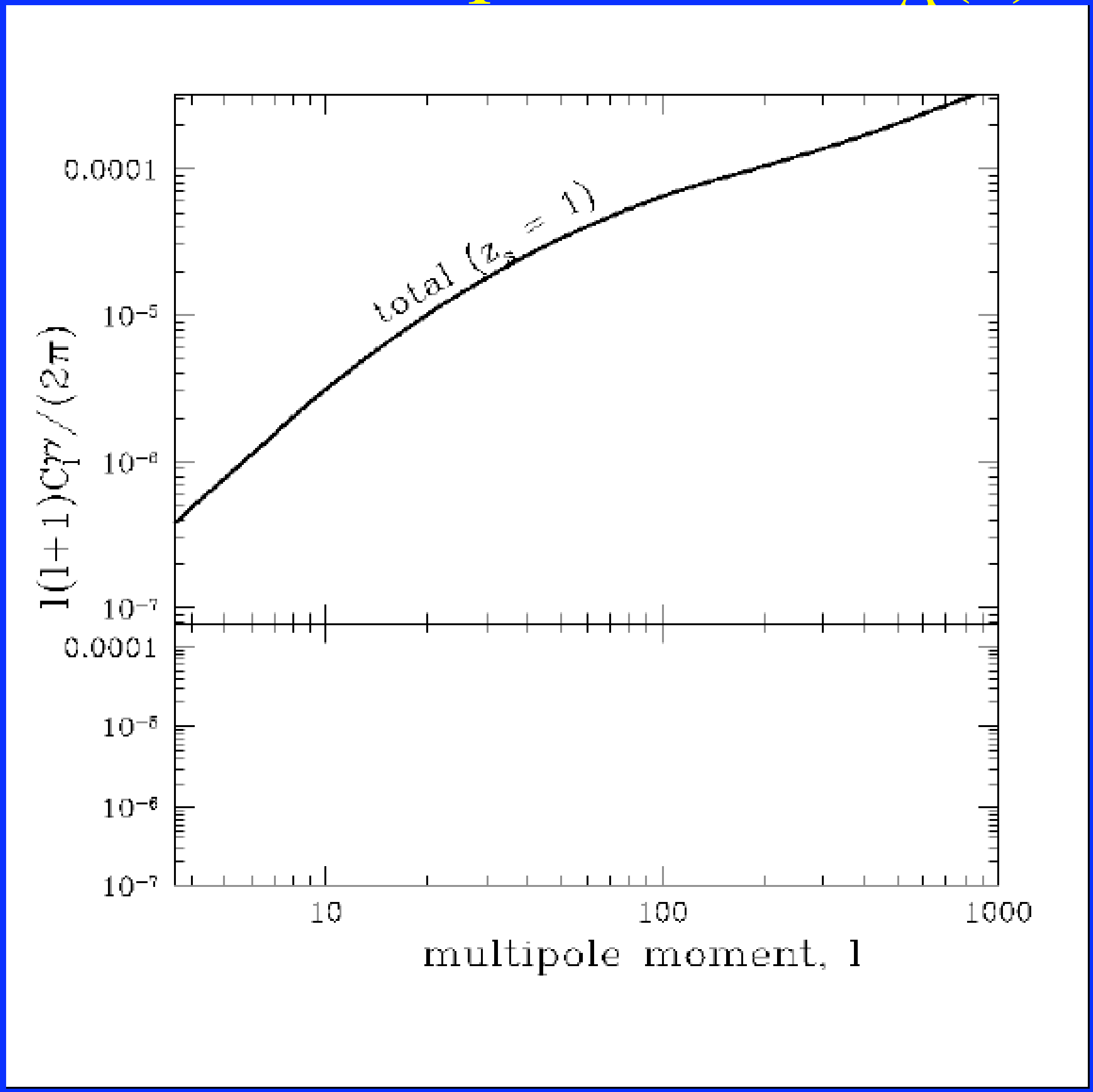
Knox, Song & Tyson (2005)



With the parameters of the high-z Universe pinned down by Planck, only thing left to measure is g(z) and $D_A(z)$ (here called r(z)) in the dark energy-dominated era. They can both be reconstructed from tomographic cosmic shear data.

D.E. constraints come almost entirely from $D_A(z)$ constraints (Simpson & Bridle '04, KST05) .

Dependence of Shear power on $D_A(z)$ and $g(z)$



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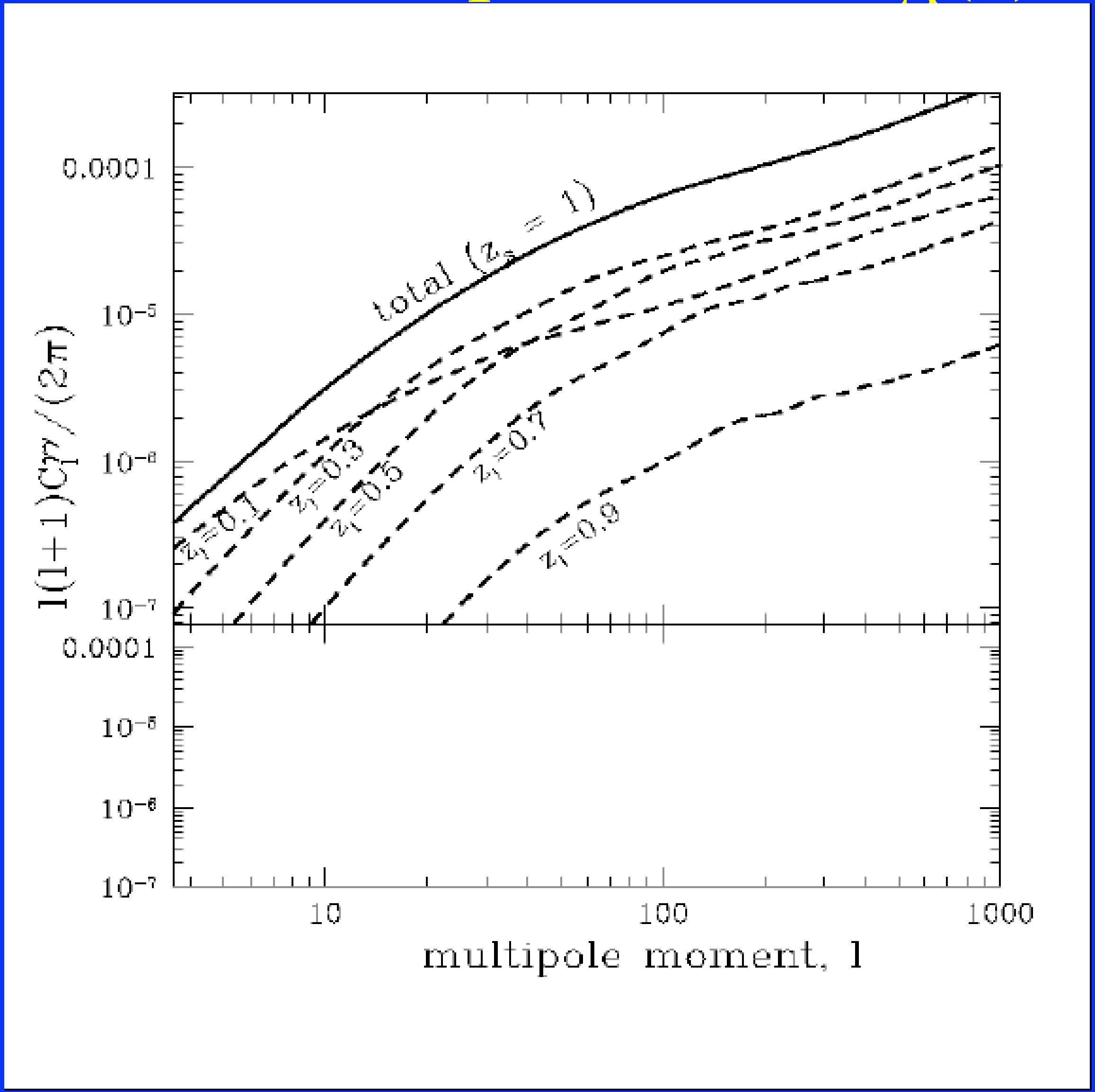

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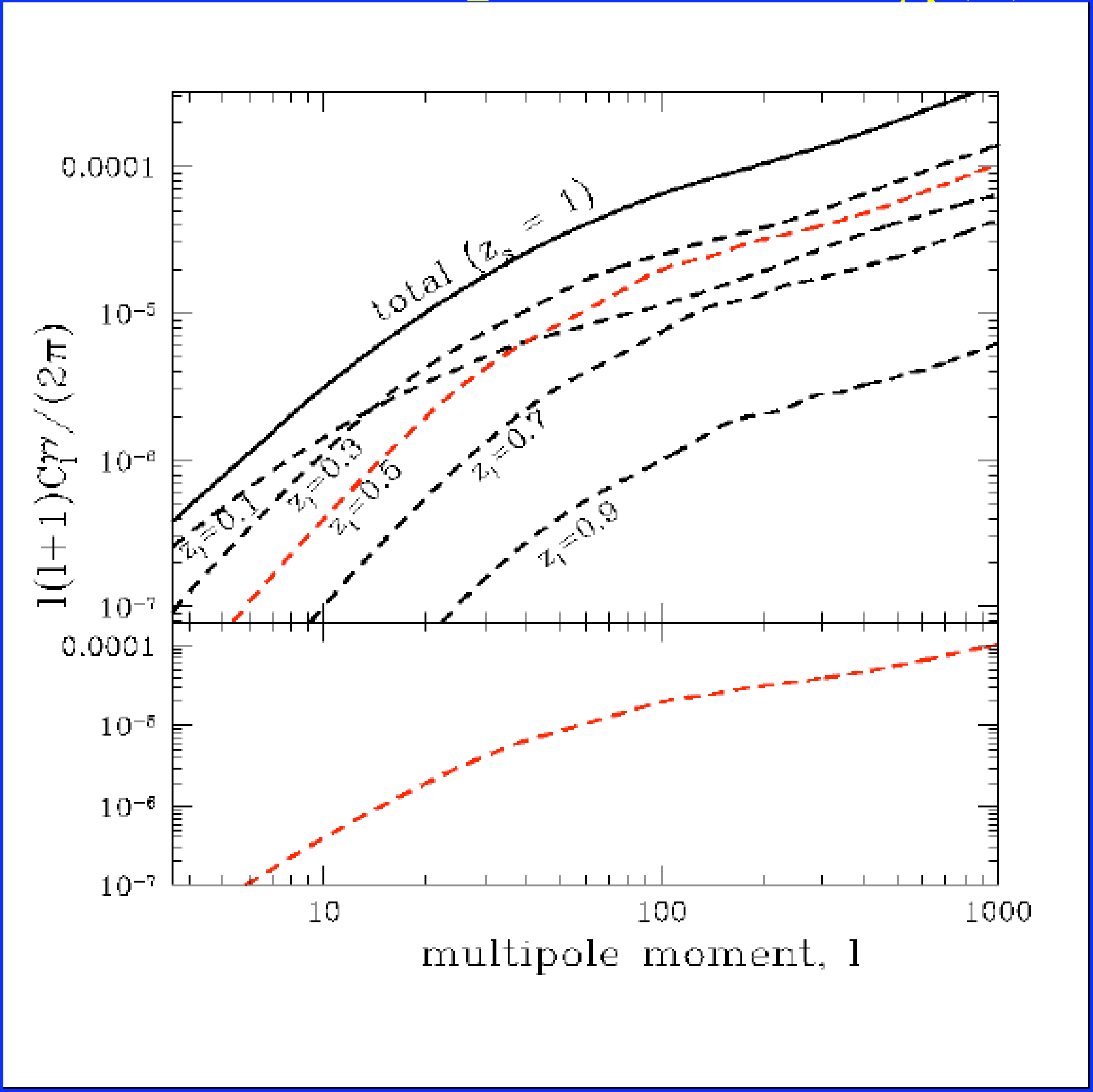
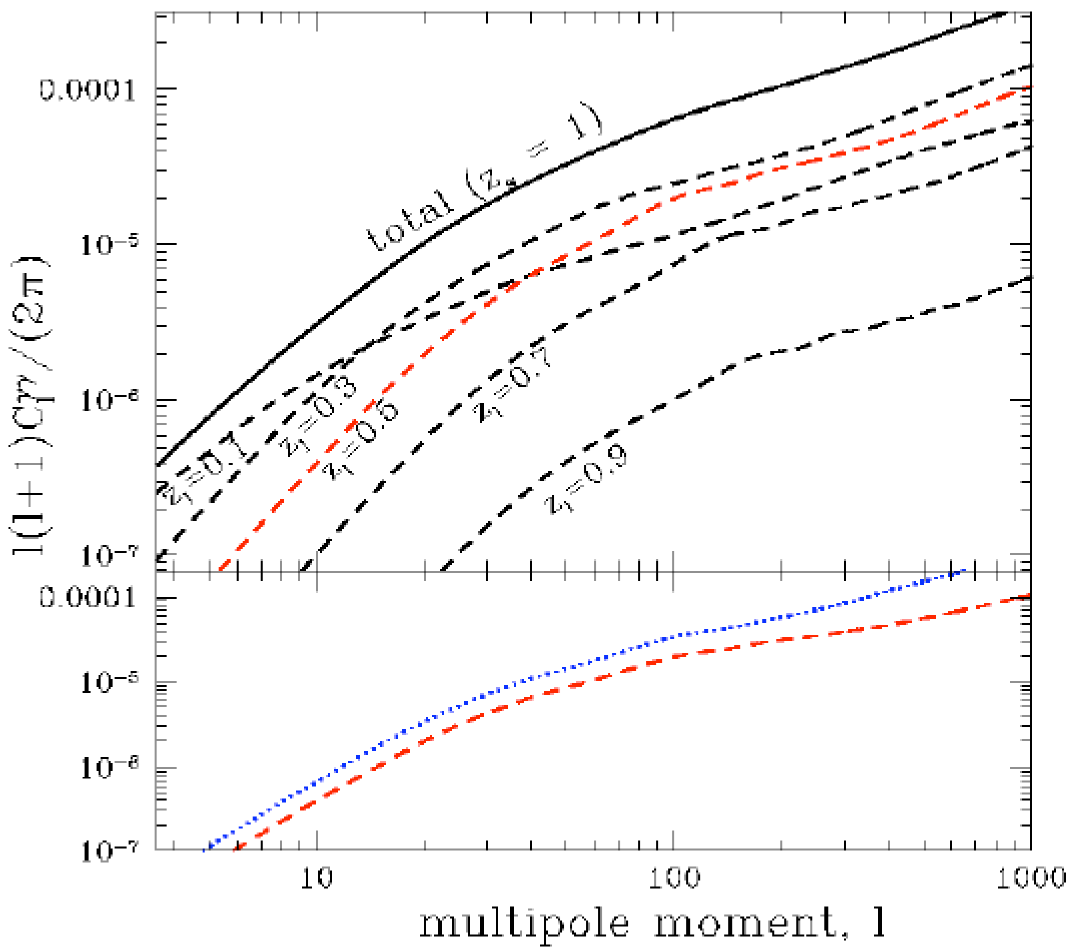
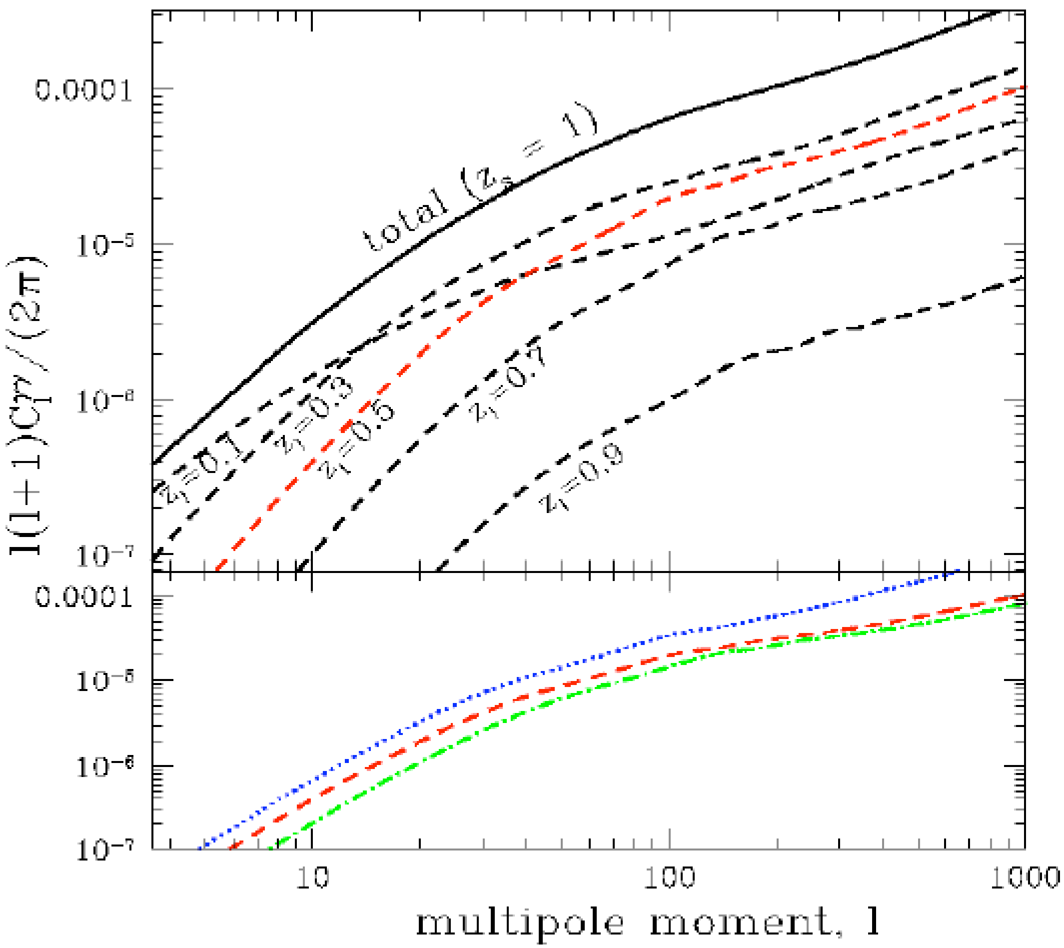


Dependence of Shear power on $D_A(z)$ and g(z)



Increasing g(z=0.5) by 30%

Dependence of Shear power on $D_A(z)$ and g(z)

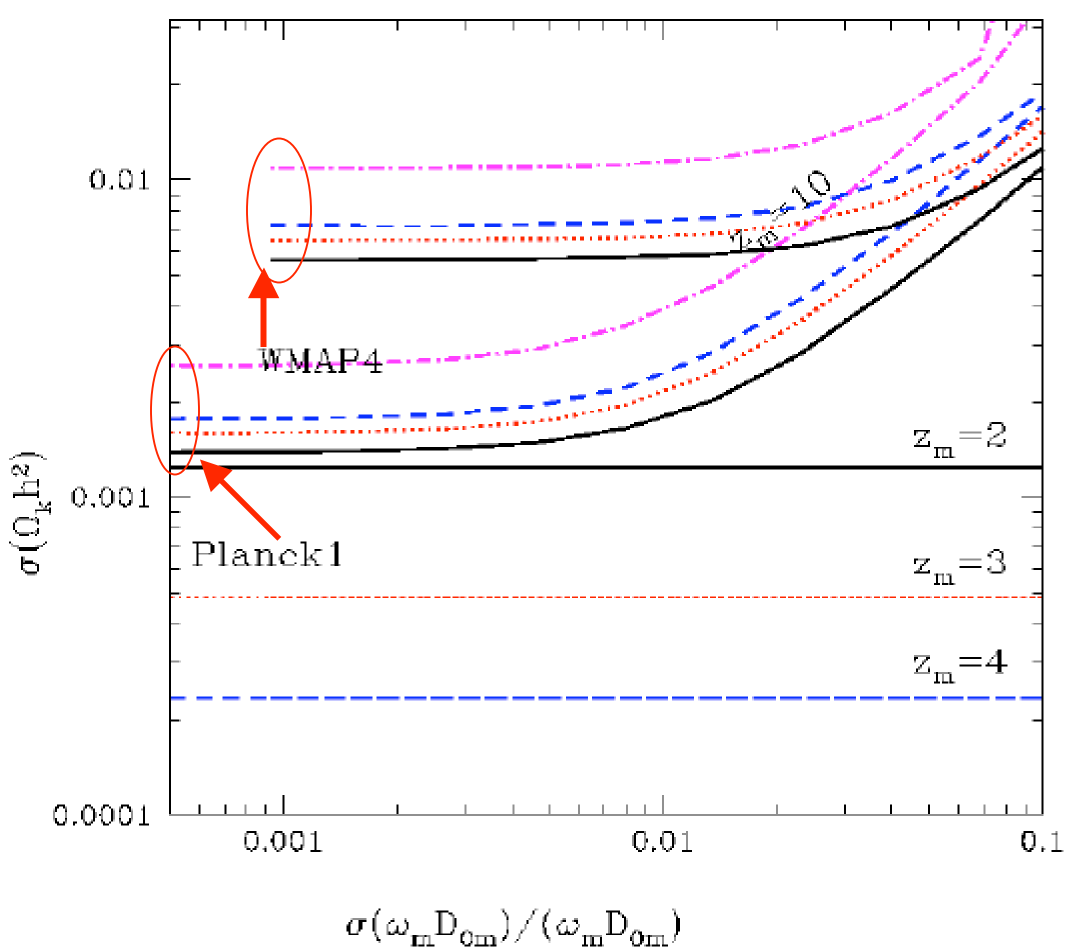


Increasing g(z=0.5) by 30%

Increasing $D_A(z)$

Curvature Error Given Error on $D_{OM}\omega_m$

$$\Omega_k h^2 = 6(h/H_0)^2\omega_m^2 (D_{OL}\omega_m - (D_{OM}\omega_m + l_{ML}\omega_m))/((D_{OL}\omega_m)^3 - (D_{OM}\omega_m)^3)$$



Limit of perfect $D_{OM}\ \omega_m$: Cancellation no longer as good between $D_{OL}\omega_m$ and $l_{ML}\omega_m$

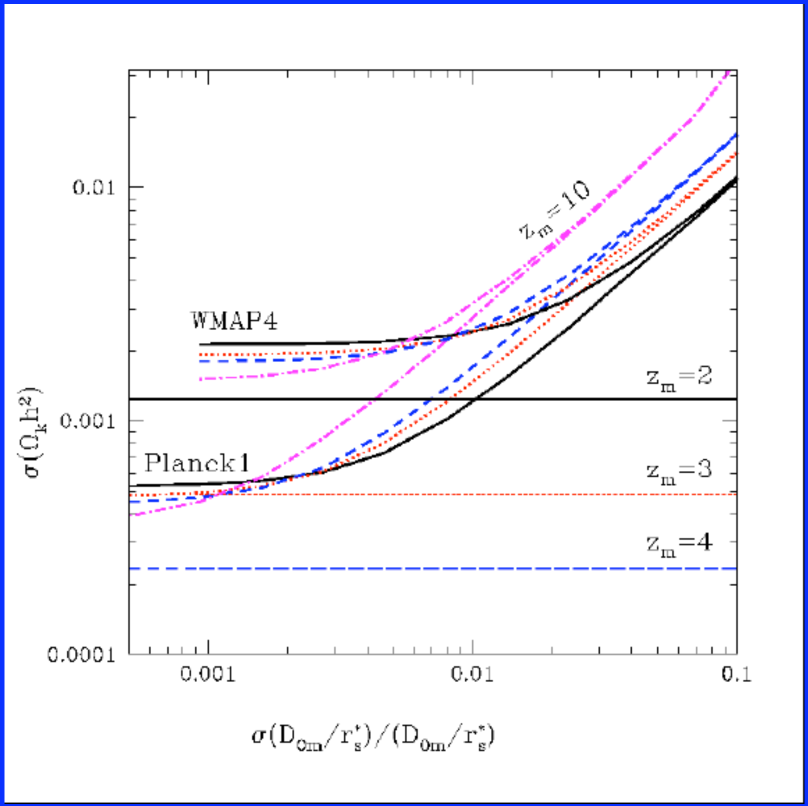
We do significantly worse here than in pure distance measurement case or in baryon oscillation case.

Note on Robustness of $\Omega_k h^2$ from BAO

- CMB acoustic peak morphology affected by evolution of gravitational potentials ➔ constrains ρ_m/ρ_{rad} and therefore ρ_m *if we know radiation content*.
- Independent of radiation content CMB robustly constrains $\rho_m^{1/2}\, r_s$.
- Since BAO constrain D_A/r_s and we know $\rho_m^{1/2}\, r_s$ we actually learn $D_A\, \rho_m^{1/2}$ (Eisenstein & White (2004))
- $\Omega_k\, h^2 / D_{OL}\omega_m^{1/2} - (D_{OM}\, \omega_m^{1/2} + \underbrace{l_{ML}\omega_m^{1/2}}_{\text{Has no dependence on cosmological parameters!}})$

What would a detection at 10^{-3} level possibly mean?

- Inflation did not happen (but then what did that leaves small curvature?)
- Inflation occurred and ended with bubble nucleation followed by ~ 60 e-folds of slow-roll. [Very fine-tuned!]
- Extra fluctuation power on super-horizon scales.



Another Way to Measure Mean Curvature

Bernstein (2005)

It's always true that

$$r_{AC} - (r_{AB} + r_{BC}) = 0$$

where A is the origin.

It's also true that $D_{AC} = r_{AC}$ and $D_{AB} = r_{AB}$,

but D_{BC} is *not* equal to r_{BC}

In fact,

$$D_{AC} - (D_{AB} + D_{BC}) / \Omega_k$$

WL is sensitive to all three distances. BAO can help.

Summary

- Zero mean curvature is a robust prediction of inflation worth rigorous checking.
- Uncertainty about dark energy limits our current knowledge of the mean curvature.
- Measurement of distances into the matter-dominated era will greatly reduce the dark energy model-dependence of any curvature determination.

WMAP

- Ωtotal=1

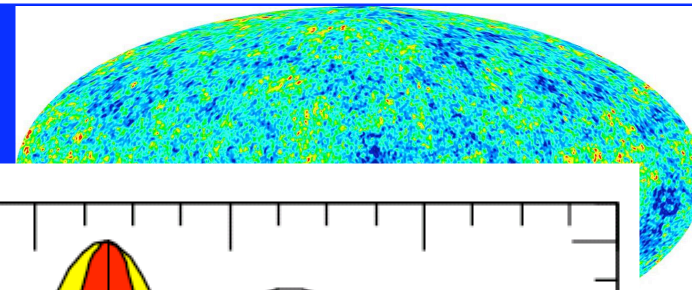
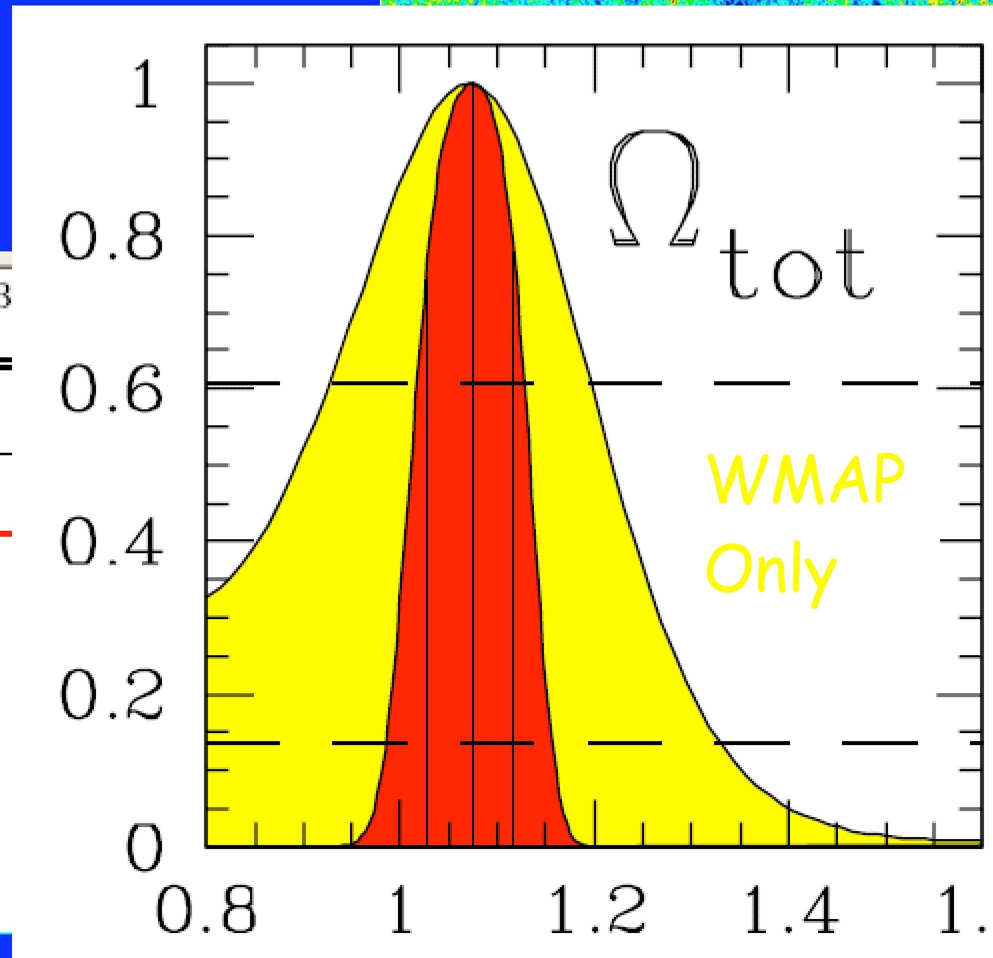


Table 3. "B

Description
Total density
Equation of state of quintessence
Dark energy density
Baryon density
Baryon density
Baryon density (cm^{-3})
Matter density
Matter density
Light neutrino density



WMAP
+ SDSS

Tegmark et al

astro-ph/0310723

Bennett et al Feb 11 '03